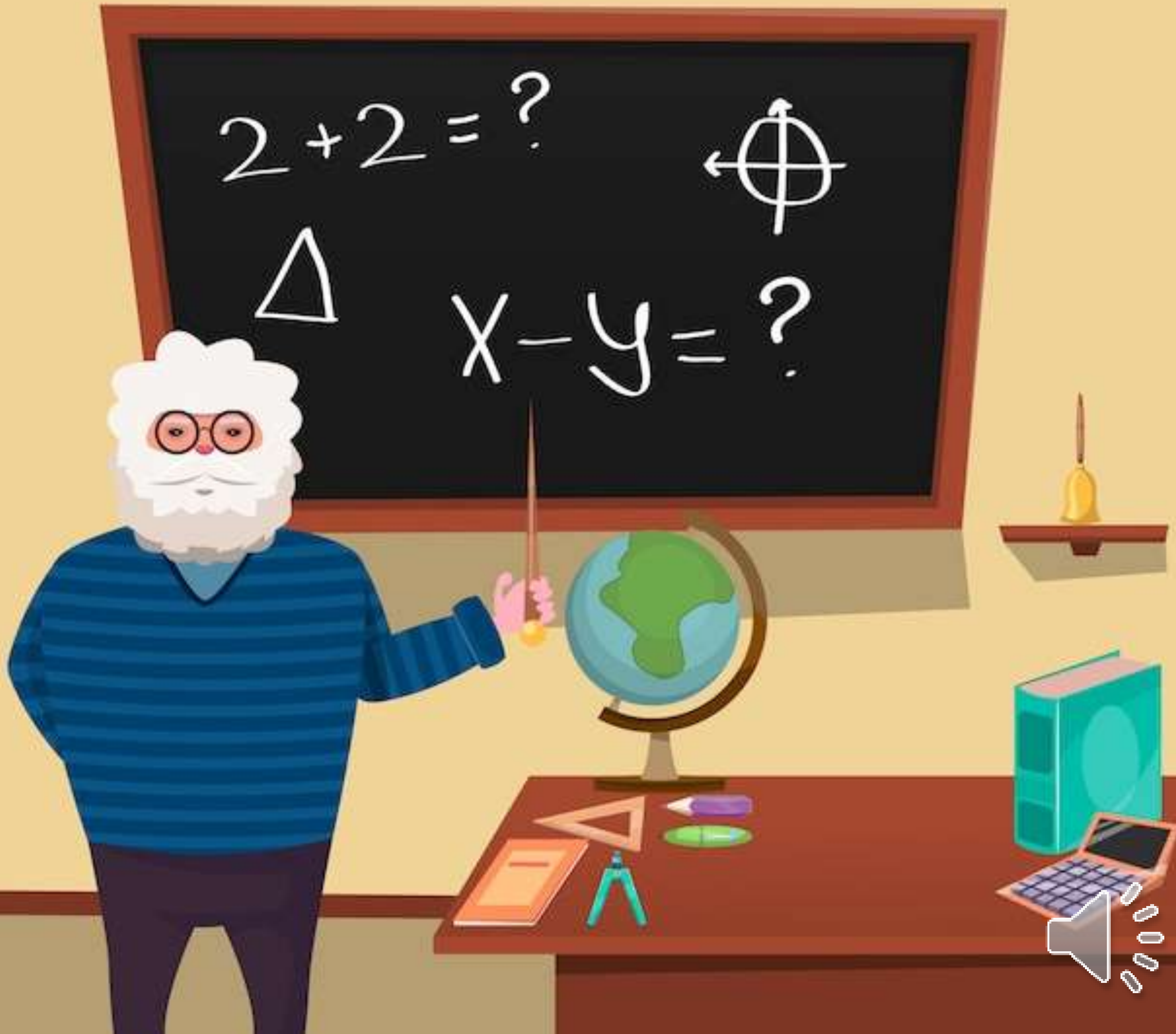
Algebraic Expressions

Part 2



Identical polynomials

Two polynomials are identical if their coefficients corresponding to like monomials (of the same degree) are equal.

Example:

The polynomials $P(x) = \frac{1}{2}x(2x - 4)$ and $Q(x) = x^2 - 2x$ are identical since:

$$P(x) = \frac{1}{2}x(2x - 4) = \left(\frac{1}{2}x\right)(2x) - \left(\frac{1}{2}x\right)(4) = x^2 - 2x = Q(x)$$

Polynomials identical to zero

A polynomial is identical to zero if all the coefficients are equal to zero.

Example:

The polynomial $P(x) = x(x-3) + x^2 - 2x(x+1) + 5x$ is identical to zero since:

$$\begin{aligned} P(x) &= x(x-3) + x^2 - 2x(x+1) + 5x \\ &= x^2 - 3x + x^2 - 2x^2 - 2x + 5x \\ &= 0x^2 + 0x = 0 \end{aligned}$$

By expanding (developing) the factors

Application #1

Consider the polynomial $P(x) = (2a - 1)x^2 + (3 - b)x + \frac{1}{2}(2 - 5c)$.

Determine a, b and c so that:

1) $P(x)$ is identical to zero.
2) $P(x)$is identical to $Q(x) = 3x - 1$

1) $P(x) = 0$ so,

$2a - 1 = 0$ and $3 - b = 0$ and $\frac{1}{2}(2 - 5c) = 0$

$2a = 1$

$\boxed{a = \frac{1}{2}}$

$-b = -3$

$\boxed{b = 3}$

$2 - 5c = 0$

$-5c = -2$

$c = \frac{-2}{-5}$

$\boxed{c = \frac{2}{5}}$

Application #1

Consider the polynomial $P(x) = (2a-1)x^2 + (3-b)x + \frac{1}{2}(2-5c)$.

Determine a, b and c so that:

1) $P(x)$ is identical to zero.
2) $P(x)$ is identical to $Q(x) = 3x - 1 + 0x^2$

2) $P(x) = Q(x)$ so,

$2a - 1 = 0$ and $3 - b = 3$ and $\frac{1}{2}(2-5c) = -1$

$2a = 1$

$a = \frac{1}{2}$

$-b = 3 - 3$

$-b = 0$

$b = 0$

$2 - 5c = -2$

$-5c = -2 - 2$

$-5c = -4$

$c = \frac{-4}{-5} = \frac{4}{5}$

Numerical value of a polynomial

Each time we give a value for x, we can calculate the corresponding value of the polynomial $P(x)$.

Example:
Consider the polynomial $P(x) = 6x^3 - 9x^2 + 3x + 1$

For $x = 1\,;\, P(1) = 6(1)^3 - 9(1)^2 + 3(1) + 1 = 1$

For $x = -2\,;\, P(-2) = 6(-2)^3 - 9(-2)^2 + 3(-2) + 1 = -89$

Root of a polynomial

A root a (or zero or solution) of a value which makes the polynomial zero.
We write $P(a) = 0$

Example:
Consider the polynomial $P(x) = 3x^2 + 4x - 7$

1 is a root of $P(x)$ since $P(1) = 3(1)^2 + 4(1) - 7 = 0$

$-\frac{7}{3}$ is a root of $P(x)$ since $P\left(-\frac{7}{3}\right) = 3\left(-\frac{7}{3}\right)^2 + 4\left(-\frac{7}{3}\right) - 7 = 0$

-1 is not a root of $P(x)$ since $P(-1) = 3(-1)^2 + 4(-1) - 7 = -8 \neq 0$

Application #2

Consider the polynomial $P(x) = 2x^2 - 7x + 5$

1) Calculate the value of $P(x)$ for $x = -1$.
2) Show that $\frac{5}{2}$ is a root of $P(x)$.

1) $P(-1) = 2(-1)^2 - 7(-1) + 5 = 2 + 7 + 5 = 14$
2) $P\left(\frac{5}{2}\right) = 2\left(\frac{5}{2}\right)^2 - 7\left(\frac{5}{2}\right) + 5 = \frac{25}{2} - \frac{35}{2} + 5 = 0$ so $\frac{5}{2}$ is a root of $P(x)$.

Application #3

Consider the polynomial $P(x) = ax^2 + (a-1)x + 1$.
Determine the value of a so that 2 is a root of $P(x)$.

2 is a root of P(x)

So, $P(2) = 0$

$a(2)^2 + (a-1)(2) + 1 = 0$

$4a + 2a - 2 + 1 = 0$

$6a - 1 = 0$

$6a = 1$

$a = \frac{1}{6}$

Application #4

Calculate the roots of the following polynomials:

1) $P(x) = 2x + 3$
2) $P(x) = 4 - 2x$
3) $P(x) = x(2x + 5)$

Recall that:
If $a \times b = 0$, then $a = 0$ or $b = 0$

1) $P(x) = 0$

$2x + 3 = 0$

$2x = -3$

$x = -\frac{3}{2}$

2) $P(x) = 0$

$4 - 2x = 0$

$-2x = -4$

$x = \frac{-4}{-2}$

$x = 2$

3) $x(2x + 5) = 0$

$x = 0$ or $2x + 5 = 0$

$2x = -5$

$x = -\frac{5}{2}$

Time for practice

Consider the polynomials $P(x) = 2x^3 - 4x^2 + x + 1$ and $Q(x) = (a^2 + 1)x^2 + 2ax + 4a - 4$.

1) Calculate $P(-2)$.
2) Is 1 a root of $P(x)$? Justify.
3) Find the value of a so that:
   a) $Q(x)$ is identical to zero.
   b) 0 is a root of $Q(x)$.

1) $P(-2) = -33$
2) Yes, since $P(1) = 0$
3) a) a doesn't exist.
   b) $a = 1$